

The Pion Electroproduction in the $\Delta(1232)$ Region

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Abstract

The amplitudes of the pion electroproduction from the nucleon are derived up to $\mathcal{O}(\epsilon^3)$ in the framework of chiral effective theory including pions, nucleons and $\Delta(1232)$ isobars as explicit degrees of freedom. The Q^2 evolutions of the weighted integrals of amplitudes are presented and the predictive power of the method proposed in the previous study of pion photoproduction [1] is shown.

1 Introduction

With the advent of the new generation of high intensity, high duty-factor electron accelerators as MAMI(Mainz), ELSA(Bonn) and Jefferson Lab, as well as modern laser backscattering facilities at LEGS(Brookhaven) and GRAAL(Grenoble), a great amount of precise data probing the structure of the nucleon at low energy has become available or is expected in the near future. Most of these data are from scattering processes with either the real or virtual photons or with pions. These different processes should not be treated separately since they are intimately related. For example the Fermi-Watson theorem [2] relates the phase shift between the pion-nucleon scattering and the pion photo- and electroproduction. In order to understand the properties of the nucleon through recent data, a consistent framework to describe these physical processes is essential.

In the low energy regime chiral symmetry largely governs the dynamics of the pions and nucleons by severely restricting the interactions between them; chiral symmetry ought to play a crucial role in the analysis of various experimental data of the $\gamma N\pi$ system. It turns out that Heavy Baryon Chiral Perturbation Theory (HBChPT) becomes a very efficient and powerful tool to study the behavior of $N\pi$ system in the low energy region because HBChPT automatically satisfies the constraint of chiral symmetry at the leading order and provides a systematic way of including the effects of finite quark mass and other sectors at higher energies. Many calculations of HBChPT on the threshold were done and most were proven successful [3].

However the validity of the original HBChPT beyond the threshold is threatened by the existence of the nucleon excited states, *i.e.*, the resonances. In the original HBChPT such states are integrated out and their effects are replaced by the finite piece of counterterms, therefore the direct connection is lost. Phenomenologically this is a reasonable scheme so long as these resonances are heavy compared to the energy scale of interest. However for the case of $\Delta(1232)$ resonance it is questionable because $\Delta(1232)$ is light and strongly couples to the $N\pi$ system. This is consistent with Large N_c QCD which requires a light, strongly coupled $I=J=3/2$ state [4, 5]. Therefore it is sensible to include the $\Delta(1232)$ as an explicit degree of freedom in the effective chiral Lagrangian applied in the $\Delta(1232)$ region. This phenomenological extension of HBChPT is presumably a reasonable and consistent scheme to investigate the processes such as $\gamma N \rightarrow \pi N$, $\pi N \rightarrow \pi N$ and $\gamma N \rightarrow \gamma N$ in the $\Delta(1232)$ region [6, 7, 8, 9], and the relations between these processes are reflected by the fact that they share many of the counterterms.

Resonances themselves are the subjects of intense interest and study. There are tremendous amount of activities, both theoretical and experimental, trying to extract the information about the $\Delta(1232) \rightarrow N$ electromagnetic transition; it is relevant for models that made specific statements about the internal wave functions of baryons. For instance, the most naive constituent models predicted the ratio of E2/M1 and C2/M1 to be both zero because the nucleon and Δ isobars are perfectly spherical since the spatial parts of their wavefunc-

tions are both ground states. The one-gluon-exchange tensor forces only gives very small values of $E2/M1$ and $C2/M1$, and different baryon models give different values generated by different mechanisms. Recently the new $p(\vec{e}, e'\vec{p})\pi^0$ experiments at MAMI and MIT/Bates, and those planned at CEBAF, have raised the interest in the theoretical calculations of the amplitudes for electroproduction of low lying baryon resonances and it was expected that the new experimental effort would improve our understanding on the $N \rightarrow \Delta$ transition [10, 11, 12, 13, 14, 15].

However, the new data strongly suggests that nonresonant amplitudes contribute significantly both in the longitudinal and transverse channels [10]. Then the interpretation of the experimentally determined R_{EM} and R_{SM} is severely complicated by the presence of processes which are coherent with the $\Delta(1232)$ resonance. These processes give rise to additional quadrupole amplitudes in the invariant mass region of interest and contaminate previous R_{EM} and R_{SM} , so that the extraction of information about $\Delta(1232)$ turns out to be more difficult than expected. It was also demonstrated that all available calculations exhibit deviation from the data. Thus an improved theoretical framework which can describe both resonant and nonresonant amplitudes simultaneously is mandated. The approach based on the effective chiral Lagrangian including $\Delta(1232)$ is a promising choice because we made no assumption on the magnitudes of the background contributions and the $\Delta(1232)$ isobars are treated explicitly with the nucleons.

Actually, it has already been shown that there is an intrinsic theoretical difficulty to separate the resonant contribution with the background part due to the ambiguity of unitarity [16]. The approach based on HBChPT certainly is free of this kind of ambiguity since HBChPT is unitary order by order. But it turns out another ambiguity emerges: the $\gamma N \Delta$ coupling constants have to absorb the divergences generated from the loop diagrams and be renormalized infinitely. Although final amplitudes do not depend on the renormalization scheme, the separation between the resonant and the background part does. Since there is no on-shell Δ isobar available in the lab, this kind of separation becomes arbitrary. Therefore any statement about $\Delta(1232)$ resonance in our framework becomes scheme dependent. Therefore instead of the $N \rightarrow \Delta$ transition we calculated the amplitudes of electroproduction of pions in the $\Delta(1232)$ region which in principle can be measured without any ambiguity.

Unfortunately the procedure of extraction of the $\gamma N \Delta$ coupling constants would not be straightforward. In our previous study on the pion photoproduction, we pointed out that the results of HBChPT could not directly compare with experimental data because in the $\Delta(1232)$ region the straightforward power counting scheme breaks down as some amplitudes become uncontrollable around the $\Delta(1232)$ pole. One natural way to cure it is to put the self energy of $\Delta(1232)$ in the propagator, then the Δ pole is removed from the real axis and the δ function in the imaginary part of the propagator is also smoothed. However this manipulation makes power counting unreliable, if not impossible. Such a manipulation is not allowed if the formal structure of the power counting scheme is required. This appears to be true in our case because the smallness of R_{EM} and R_{SM} implies that the $\gamma N \Delta$ vertices we are studying is very weak, and therefore the error of theoretical calculation is crucial for the

verification. Thus one reasonable way to compare the HBChPT results with experimental data is via weighted integrals of the amplitudes through the $\Delta(1232)$ region. So we cannot make predictions on the physical observables directly, but only on the integrated quantities about the amplitudes of the physical processes. It becomes the main limitation of our approach.

The same limitation remains in the case of electroproduction of pions. The $\Delta(1232)$ pole also exists in electroproduction amplitudes except in S-wave. Again we cannot explicitly write down any Q^2 dependences of the physical observables. Unlike some model calculation, our approach cannot be generalized to the high Q^2 region without any modifications. But it was found that in HBChPT the photo- and electroproduction of pions are both determined by almost the same set of counterterms. In other words, once the counterterms are fit by the data of photoproduction, then the Q^2 evolution of the amplitudes of electroproduction is almost fixed in HBChPT, and the Q^2 dependences of weighted integrals of those multipoles still provide a very good testing ground for this chiral effective theory.

This article is organized as following: In Sec. 2 the general formalism is briefly sketched. Section 3 discusses the renormalization. The results of weighted integrals are given in Sec 4. We summarize and provide our perspective for further efforts in Sec 5.

2 Formulation

Heavy Baryon Chiral Perturbation Theory (HBChPT) has been quite successful for scattering processes off a single nucleon near threshold [3]. The extended formulation including $\Delta(1232)$ isobar as explicit degrees of freedom is also well developed. The basic idea is to treat $\Delta = m_\Delta - m_N$ as a light scale as m_π and expand the S matrix by $\epsilon = \{\frac{m_\pi}{\Lambda}, \frac{p}{\Lambda}, \frac{\Delta}{\Lambda}\}$, where Λ is the heavy scale typically of order such as $4\pi f_\pi$ or M_p . For details we refer the readers to extensive literature of reviews. [8, 9, 17]

The procedure for the calculation of pion electroproduction in this chiral effective theory is similar to the one of pion photoproduction. The only new $\gamma N \Delta$ vertex is:

$$\frac{-G_3}{(2m_N)^2} \bar{\psi}_i^\mu \Theta_{\mu\nu}(y_1) \gamma_5 \text{Tr}(\tau^i D_\rho f_+^{\rho\nu}) \psi_N + h.c.$$

Which vanishes in the real photon case but contributes in the virtual photon case. However it turns out that its effect starts from $\mathcal{O}(\epsilon^4)$ amplitudes, therefore all the Feymann rules we need are already in our previous work [1] and the whole calculation is straightforward.

To extract the multipole results, the amplitudes are decomposed into the standard CGLN amplitudes in the πN c.m. frame [19]:

$$\begin{aligned} \mathcal{T} = & i(\vec{\sigma} \cdot \vec{\epsilon})\mathcal{T}_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k} \times \vec{\epsilon})\mathcal{T}_2 + i(\vec{\sigma} \cdot \hat{k})(\hat{q} \cdot \vec{\epsilon})\mathcal{T}_3 + i(\vec{\sigma} \cdot \hat{q})(\vec{\epsilon} \cdot \hat{q})\mathcal{T}_4 \\ & + i(\vec{\sigma} \cdot \hat{k})(\hat{k} \cdot \vec{\epsilon})\mathcal{T}_5 + i(\vec{\sigma} \cdot \hat{q})(\hat{k} \cdot \vec{\epsilon})\mathcal{T}_6 - i(\vec{\sigma} \cdot \hat{q})\epsilon_0\mathcal{T}_7 - i(\vec{\sigma} \cdot \hat{k})\epsilon_0\mathcal{T}_8. \end{aligned} \quad (1)$$

Note \mathcal{T}_5 , \mathcal{T}_6 , \mathcal{T}_7 and \mathcal{T}_8 vanish in pion photoproduction. Due to the current conservation there are two relations:

$$|\vec{k}|\mathcal{T}_5 = k_0\mathcal{T}_8; \quad |\vec{k}|\mathcal{T}_6 = k_0\mathcal{T}_7.$$

So only six amplitudes are independent. Since our gauge condition is $\epsilon \cdot v=0$ and we choose $v_\mu=(1, \vec{0})$, the amplitudes are simplified as:

$$\begin{aligned} \mathcal{T} = & i(\vec{\sigma} \cdot \vec{\epsilon})\mathcal{T}_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot \hat{k} \times \vec{\epsilon})\mathcal{T}_2 + i(\vec{\sigma} \cdot \hat{k})(\hat{q} \cdot \vec{\epsilon})\mathcal{T}_3 + i(\vec{\sigma} \cdot \hat{q})(\vec{\epsilon} \cdot \hat{q})\mathcal{T}_4 \\ & + i(\vec{\sigma} \cdot \hat{k})(\hat{k} \cdot \vec{\epsilon})\mathcal{T}_5 + i(\vec{\sigma} \cdot \hat{q})(\hat{k} \cdot \vec{\epsilon})\mathcal{T}_6. \end{aligned} \quad (2)$$

The amplitudes are usually expressed in terms of three types of multipoles : electric($E_{l\pm}$), magnetic($M_{l\pm}$) and longitudinal ($L_{l\pm}$), with pion angular momentum l and total momentum $j = l \pm 1/2$. They can be calculated by inverting the following relations:

$$\mathcal{T}_1 = \sum_{l \geq 0} [(lM_{l+} + E_{l+})P'_{l+1} + [(l+1)M_{l-} + E_{l-}]P'_{l-1}], \quad (3)$$

$$\mathcal{T}_2 = \sum_{l \geq 1} [(l+1)M_{l+} + lM_{l-}]P'_l, \quad (4)$$

$$\mathcal{T}_3 = \sum_{l \geq 1} [(E_{l+} - M_{l+})P''_{l+1} + (E_{l-} + M_{l-})P''_{l-1}], \quad (5)$$

$$\mathcal{T}_4 = \sum_{l \geq 2} (M_{l+} - E_{l+} - M_{l-} - E_{l-})P''_l, \quad (6)$$

$$\mathcal{T}_5 = \sum_{l \geq 0} [(l+1)L_{l+}P'_{l+1} - lL_{l-}P'_{l-1}], \quad (7)$$

$$\mathcal{T}_6 = \sum_{l \geq 1} [(lL_{l-} - (l+1)L_{l+})P'_l]. \quad (8)$$

Here P'_l are derivatives of Legendre polynomials. Note that in the literature the longitudinal transitions are often described by $S_{l\pm}$ scalar multipoles which correspond to the multipole decomposition of the amplitudes \mathcal{T}_7 , \mathcal{T}_8 . They are connected with the longitudinal ones by $S_{l\pm} = |\vec{k}|L_{l\pm}/k_0$.

All of the observables are products of these amplitudes. In general there are 16 different polarization observables for the reaction with real photons.[20] For the virtual photon we have four additional ones due to longitudinal amplitudes and 16 observables due to longitudinal-transverse interference. Thus there are 36 observables for pion electroproduction. In view of the great number of possible polarization observables it is natural to ask which set of observables can be, in principle, a complete determination of all amplitudes. Naively it may

be argued that any set of 11 observables should suffice to determine all amplitudes because there are six independent complex variables. But one overall phase is undetermined. However all observables are the products of amplitudes therefore the discrete ambiguity prevents us to take an arbitrary ones but properly chosen set of observables to satisfy some criterion [20].

3 Renormalization

Our calculation contains several $N\pi$ and $\Delta\pi$ loop diagrams which are regularized by dimensional regularization. Their divergences must be absorbed by the counterterms b_i of $\mathcal{L}_N^{(3)}$ or G_i of $\mathcal{L}_{N\Delta}^{(3)}$. The renormalization of the amplitudes of pion photoproduction was discussed in detail in [1]. In the neutral pion electroproduction no new counterterms are needed. In the charged pion electroproduction, two new parameters b_7 , and b_{23} emerges. They are the coefficients of the following counterterms in $\mathcal{L}_{\pi N}^{(3)}$ [21] respectively:

$$\mathcal{O}_7 = [D^\mu, f_{+\mu\nu}]v^\nu, \quad \mathcal{O}_{23} = S^\mu [D^\nu, f_{-\mu\nu}].$$

Furthermore it was found that

$$\beta_7 = \frac{1}{6} + \frac{5}{6}g_A^2 - \frac{20}{27}g_{\pi\Delta N}^2. \quad (9)$$

Here

$$b_i = b_i^r(\mu) + (4\pi)^2\beta_i L$$

$$L \equiv \frac{\mu^{d-4}}{(4\pi)^2} \left(\frac{1}{d-4} - \frac{1}{2} [\ln(4\pi) + 1 + \Gamma'(1)] \right).$$

b_7^r is related to the electric mean square charge radii of the proton. Consider the nucleon matrix element of the isovector component of the quark vector current:

$$\langle N(p') | \bar{q} \gamma_\mu \frac{\tau^a}{2} q | N(p) \rangle = \bar{u}(p') [\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_p} F_2^V(q^2)] \frac{\tau^a}{2} u(p),$$

with $q = p' - p$.

$$\langle r^2 \rangle = 6 \frac{dF_1^V(q^2)}{dq^2} \Big|_{q^2=0}.$$

The relation was given by [24]:

$$\begin{aligned} \langle r^2 \rangle_1 = & -\frac{1}{(4\pi F_\pi)^2} \{ 1 + 7g_A^2 + (10g_A^2 + 2) \ln(\frac{m_\pi}{\mu}) \} - \frac{12b_7^r(\mu)}{(4\pi F_\pi)^2} \\ & + \frac{g_\pi^2 \Delta N}{54\pi^2 F_\pi^2} \{ 26 + 30 \ln(\frac{m_\pi}{\mu}) + 30 \frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}} \ln[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}] \}. \end{aligned} \quad (10)$$

In the large N_c limit, β_7 is simply $\frac{1}{6}$ [22] due to the same reason for simplification of β_{17} [1].

On the other hand, b_{23} absorbs no divergence and its value is related to the axial mean square radius:

$$b_{23} = \frac{g_A}{6} \langle r_A^2 \rangle. \quad (11)$$

The data from (anti)neutrino-proton scattering gives $(4\pi F_\pi)^2 b_{23} = 3.08 \pm 0.27$. Besides the counterterms in $\mathcal{L}_{\pi N}$, the diagrams of t channel also involves the counterterms in $\mathcal{L}_{\pi\pi}$:

$$\mathcal{L}_{\pi\pi}^{(4)} = \frac{l_3}{16} \text{Tr}(\chi_+^2) + \frac{l_4}{16} \{ 2\text{Tr}(D_\mu U D^\mu U^\dagger \text{Tr}(\chi_-)^2 + 2\text{Tr}(\chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger) - 4\text{Tr}(\chi^\dagger \chi) - (\text{Tr}(\chi_-)^2) \} + i \frac{l_6}{2} \text{Tr}([u^\mu, u^\nu]) f_{\mu\nu}^+ + \dots \quad (12)$$

l_3 and l_4 appear in the chiral correction of pion decay constant and pion mass respectively:

$$m_\pi^2 = m_0^2 + \frac{m_0^4}{F_\pi^2} (2l_3^r(\mu) + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu}). \quad (13)$$

$$F_\pi = F_0 + \frac{m_0^2}{F_0} (l_4^r(\mu) - \frac{1}{8\pi^2} \ln \frac{m_\pi}{\mu}). \quad (14)$$

l_6 only emerges at processes of pion electroproduction. It also absorbs the divergence:

$$l_6 = -\frac{L}{6} + l_6^r(\mu). \quad (15)$$

Its finite part can be fixed by the empirical value of the pion mean square charge radius:

$$\langle r^2 \rangle_\pi = -\frac{1}{8\pi^2 F_\pi^2} (\ln \frac{m_\pi}{\mu} - \frac{12}{F_\pi^2} l_6^r(\mu)). \quad (16)$$

Using the empirical values $\langle r^2 \rangle_\pi = 0.439 \text{ fm}^2$ [23], we have $l_6(\mu = 1\text{Gev}) = 6.6 \times 10^{-3}$. Therefore the pion electroproduction at this order shares the same set of unknown parameters and introduce none which cannot be independently fit by other processes.

4 Result and Discussion

The results contain the s-channel $\Delta(1232)$ pole. As mentioned before the only known way to keep both unitarity and the power counting scheme is to calculate the weighted integrals as proposed in [1]:

$$\bar{M}_{l\pm}^{(n)} = \frac{1}{m_p} \int_{m_\pi}^{E_{max}} M_{l\pm}(E) \left(\frac{E}{m_p} \right)^n dE. \quad (17)$$

The weighted integrals are parameterized as following:

$$\begin{aligned} \text{Re} \bar{P}_i^{\pi^0 P} = & e g_A \zeta_i^A + e g_A \dot{\kappa}_p \zeta_{i;B}^B + e g_A \kappa_p \zeta_{i;R}^B + e g_A (1 + \dot{\kappa}_p) \tilde{c}_1 \zeta_i^C \\ & + e g_A^3 \zeta_i^D + e g_{\pi\Delta N} \dot{G}_1 \zeta_{i;B}^E + e g_{\pi\Delta N} G_1 \zeta_{i;R}^E + e g_{\pi\Delta N} \tilde{G}_2 \zeta_i^F + e g_{\pi\Delta N} \tilde{G}_6 \zeta_i^G \\ & + e \tilde{g}_{\pi\Delta N} \dot{G}_1 \zeta_i^H + e g_{\pi\Delta N}^2 g_A \zeta_i^K + e g_{\pi\Delta N}^2 g_1 \zeta_i^L + e \tilde{b}_9 \zeta_i^M, \\ & i = 1, 2, 3. \end{aligned} \quad (18)$$

$$\begin{aligned}
\frac{1}{\pi}Im\bar{P}_i^{\pi^0 P} &= eg_A^3 \xi_i^D + eg_{\pi\Delta N} \dot{G}_1 \xi_{i;B}^E + eg_{\pi\Delta N} G_1 \xi_{i;R}^E + eg_{\pi\Delta N} \tilde{G}_2 \xi_i^F + eg_{\pi\Delta N} \tilde{G}_6 \xi_i^G \\
&\quad + e\tilde{g}_{\pi\Delta N} \dot{G}_1 \xi_i^H + eg_{\pi\Delta N}^2 g_A \xi_i^K + eg_{\pi\Delta N}^2 g_1 \xi_i^L, \\
i &= 1, 2, 3.
\end{aligned} \tag{19}$$

Here $\tilde{G}_2 = G_2 + 4G_4$, $\tilde{G}_6 = G_6 - G_4$, $\tilde{c}_1 = m_N c_1$, $\tilde{b}_9 = b_9 - b_{10} - \frac{(4\pi F_\pi)^2}{6m_N^2} g_{\pi\Delta N} \dot{G}_1 (1 + 4x + 4z + 12xz)$. $\dot{\kappa}_p$ means the parameter is taken in the limit: $\Delta \rightarrow 0$, $m_\pi \rightarrow 0$, $\frac{\Delta}{m_\pi}$ fixed. The first four terms in (18) are from tree graphs without the delta; the sixth to eleventh terms are due to tree graphs with the delta. Note that such tree graphs also contribute to the imaginary parts of amplitudes due to the delta function in $\frac{1}{E - \Delta + i\epsilon}$. The fifth term is from loop graphs without delta; the twelfth and thirteenth terms are $\Delta - \pi$ loop contributions; the last term, which only appears in P_3 , is due to the counterterms in $\mathcal{L}_{\pi NN}^{(3)}$. Note that the quantities, such as ξ_i^K, ξ_i^L are μ -dependent, however final amplitudes are independent of μ because the κ_v , G_1 , \tilde{G}_2 and \tilde{G}_6 are also μ -dependent, and compensate the ones from the loop.

Similarly the longitude multipoles L_{1+} and L_{1-} also suffer from the same s-channel $\Delta(1232)$ pole, therefore the same method is applied to them and the weighted integrals are parameterized as:

$$\begin{aligned}
Re\bar{L}_{1\pm}^{\pi^0 P} &= eg_A \zeta_\pm^A + eg_A (1 + \dot{\kappa}_p) \tilde{c}_1 \zeta_\pm^C + eg_A^3 \zeta_\pm^D + eG_{\pi\Delta N} \dot{G}_1 \zeta_{\pm;B}^E \\
&\quad + eg_{\pi\Delta N} \tilde{G}_2 \zeta_\pm^F + eg_{\pi\Delta N}^2 g_A \zeta_\pm^K + eg_{\pi\Delta N}^2 g_1 \zeta_\pm^L.
\end{aligned} \tag{20}$$

$$\begin{aligned}
\frac{1}{\pi}Im\bar{L}_{1\pm}^{\pi^0 P} &= eg_A^3 \xi_\pm^D + eg_{\pi\Delta N} \dot{G}_1 \xi_{\pm;B}^E + eg_{\pi\Delta N} \tilde{G}_2 \xi_\pm^F \\
&\quad + eg_{\pi\Delta N}^2 g_A \xi_\pm^H + eg_{\pi\Delta N}^2 g_1 \xi_\pm^K.
\end{aligned} \tag{21}$$

Note that there is no $\mathcal{O}(\epsilon^2)$ piece in L_{1+} , and the $\mathcal{O}(\epsilon^2)$ piece of L_{1-} is entirely due to the nucleon.

Here we set $\Delta = 294$ Mev and $F_\pi = 92.4$ Mev, $M_N = 938.7$ Mev, $E_{max} = 340$ Mev and $\mu = 500$ Mev. All quantities are in the unit of $10^{-4}/m_\pi$:

n	ζ_1^A	$\zeta_{1;R}^B$	$\zeta_{1;B}^R$	ζ_1^C	ζ_1^D	$\zeta_{1;R}^E$	$\zeta_{1;B}^E$	ζ_1^F	ζ_1^G	ζ_1^H	$\zeta_{1;\pi}^K$	$\zeta_{1;\Delta}^K$	$\zeta_{1;\pi}^L$	$\zeta_{1;\Delta}^L$
1	24.80	29.26	-2.31	2.83	11.34	-12.97	-4.34	0.78	-3.13	-3.13	-7.58	5.64	4.83	0
2	6.81	8.12	-0.71	0.78	3.34	-4.04	-1.14	0.31	-1.23	-1.23	-1.64	1.53	2.03	0
3	1.95	2.35	-0.22	0.22	1.01	-1.76	-0.31	0.11	0.08	0.08	-0.36	0.42	1.14	0
n	ξ_1^D	$\xi_{1;R}^E$	$\xi_{1;B}^E$	ξ_1^F	ξ_1^G	ξ_1^H	$\xi_{1;\pi}^K$	$\xi_{1;\Delta}^K$	$\xi_{1;\pi}^L$	$\xi_{1;\Delta}^L$				
1	2.51	6.64	-0.90	-0.45	1.80	1.80	-0.16	-2.19	0	-3.71				
2	0.78	1.80	-0.25	-0.12	0.49	0.49	0.03	-0.60	0	-1.00				
3	0.25	0.49	-0.07	-0.03	0.13	0.13	0.03	-0.16	0	-0.27				
n	ζ_2^A	$\zeta_{2;R}^B$	$\zeta_{2;B}^B$	ζ_2^C	ζ_2^D	$\zeta_{2;R}^E$	$\zeta_{2;B}^E$	ζ_2^F	ζ_2^G	ζ_2^H	$\zeta_{2;\pi}^K$	$\zeta_{2;\Delta}^K$	$\zeta_{2;\pi}^L$	$\zeta_{2;\Delta}^L$
1	-14.96	-29.26	2.31	-2.83	-16.18	12.97	5.83	1.57	3.13	3.13	10.16	22.22	-2.56	0
2	-4.12	-8.12	0.71	-0.78	-4.58	4.04	2.03	0.62	1.23	1.23	3.09	6.03	-1.04	0
3	-1.18	-2.35	0.22	-0.22	-1.35	1.76	0.69	0.22	-0.08	-0.08	0.96	1.64	-0.38	0
n	ξ_2^D	$\xi_{2;R}^E$	$\xi_{2;B}^E$	ξ_2^F	ξ_2^G	ξ_2^H	$\xi_{2;\pi}^K$	$\xi_{2;\Delta}^K$	$\xi_{2;\pi}^L$	$\xi_{2;\Delta}^L$				
1	6.65	-6.64	-1.80	-0.90	-1.80	-1.80	8.12	-1.92	0	1.78				
2	-2.01	-1.80	-0.49	-0.24	-0.49	-0.49	2.77	-0.52	0	0.48				
3	-0.62	-0.49	-0.14	-0.06	-0.13	-0.13	1.19	-0.14	0	0.13				
n	ζ_3^A	$\zeta_{3;R}^B$	$\zeta_{3;B}^B$	ζ_3^C	$\zeta_{3;R}^E$	$\zeta_{3;B}^E$	ζ_3^F	ζ_3^G	ζ_3^H	$\zeta_{3;\pi}^K$	$\zeta_{3;\Delta}^K$	$\zeta_{3;\pi}^L$	$\zeta_{3;\Delta}^L$	ζ_3^M
1	-0.86	4.58	-9.12	-9.49	13.20	-1.17	1.24	9.90	9.90	-6.86	-16.58	12.92	0	20.64
2	-0.30	1.31	-2.67	-2.53	6.27	-0.19	0.44	3.53	3.53	-2.32	-4.50	7.17	0	5.96
3	-0.10	0.39	-0.81	-0.71	2.46	-0.02	0.15	1.21	1.21	-0.77	-1.22	2.96	0	1.78
n	$\xi_{3;R}^E$	$\xi_{3;B}^E$	ξ_3^F	ξ_3^G	ξ_3^H	$\xi_{3;\pi}^K$	$\xi_{3;\Delta}^K$	$\xi_{3;\pi}^L$	$\xi_{3;\Delta}^L$					
1	-13.29	-0.90	-0.45	-3.36	-3.36	-4.90	2.33	0	-18.59					
2	-3.60	-0.24	-0.12	-0.91	-0.91	-1.71	0.63	0	-5.05					
3	-0.98	-0.07	-0.03	-0.25	-0.25	-0.58	0.17	0	-1.37					
n	ζ_+^A	ζ_+^D	$\zeta_{+;B}^E$	ζ_+^F	$\zeta_{+;\pi}^K$	$\zeta_{+;\Delta}^K$	$\zeta_{+;\pi}^L$	$\zeta_{+;\Delta}^L$						
1	1.64	-0.81	0.25	0.39	1.54	4.64	0.45	0						
2	0.45	-0.21	0.15	0.16	0.62	1.26	0.19	0						
3	0.13	-0.06	0.06	0.06	0.22	0.34	0.39	0						
n	ξ_+^D	$\xi_{+;B}^E$	ξ_+^F	$\xi_{+;\pi}^K$	$\xi_{+;\Delta}^K$	$\xi_{+;\pi}^L$	$\xi_{+;\Delta}^L$							
1	-0.69	-0.45	-0.23	8.60	-1.03	0	-0.33							
2	-0.21	-0.12	-0.06	2.92	-0.28	0	-0.09							
3	-0.06	-0.06	-0.02	1.02	-0.08	0	-0.02							
n	ζ_-^A	ζ_-^C	ζ_-^D	$\zeta_{-;B}^E$	ζ_-^F	$\zeta_{-;\pi}^K$	$\zeta_{-;\Delta}^K$	$\zeta_{-;\pi}^L$	$\zeta_{-;\Delta}^L$					
1	1.20	-8.44	-8.88	1.29	2.56	6.29	0	0.46	0					
2	0.29	-2.25	-2.30	0.61	0.91	1.85	0	0.17	0					
3	0.08	-0.63	-0.61	0.23	0.34	0.57	0	-0.59	0					
n	ξ_-^D	$\xi_{-;B}^E$	ξ_-^F	$\xi_{-;\pi}^K$	$\xi_{-;\Delta}^K$	$\xi_{-;\pi}^L$	$\xi_{-;\Delta}^L$							
1	-6.65	-1.36	-1.38	3.03	0.25	0	-0.28							
2	-1.98	-0.36	-0.42	0.97	0.08	0	-0.08							
3	-0.74	-0.11	-0.17	0.04	0.01	0	-0.03							

Table 1: $Q^2=0.01 \text{ (Gev/c)}^2$

n	ζ_1^A	$\zeta_{1;R}^B$	$\zeta_{1;B}^R$	ζ_1^C	ζ_1^D	$\zeta_{1;R}^E$	$\zeta_{1;B}^E$	ζ_1^F	ζ_1^G	ζ_1^H	$\zeta_{1;\pi}^K$	$\zeta_{1;\Delta}^K$	$\zeta_{1;\pi}^L$	$\zeta_{1;\Delta}^L$
1	25.27	35.73	-7.96	4.21	13.82	-16.07	-5.57	0.93	-3.70	-3.70	-9.31	2.11	7.65	0
2	6.87	9.79	-2.23	1.13	4.04	-5.89	-1.49	0.35	-1.39	-1.39	-1.96	0.54	2.99	0
3	1.95	2.80	-0.65	0.32	1.22	-2.03	-0.42	0.12	0.10	0.10	-0.42	0.14	1.06	0
n	ξ_1^D	$\xi_{1;R}^E$	$\xi_{1;B}^E$	ξ_1^F	ξ_1^G	ξ_1^H	$\xi_{1;\pi}^K$	$\xi_{1;\Delta}^K$	$\xi_{1;\pi}^L$	$\xi_{1;\Delta}^L$				
1	3.69	7.27	-0.94	-0.47	1.88	1.88	-0.74	-2.83	0	-4.74				
2	1.11	1.88	-0.24	-0.12	0.49	0.49	-0.21	-0.73	0	-1.23				
3	0.34	0.49	-0.06	-0.03	0.13	0.13	-0.06	-0.19	0	-0.32				
n	ζ_2^A	$\zeta_{2;R}^B$	$\zeta_{2;B}^B$	ζ_2^C	ζ_2^D	$\zeta_{2;R}^E$	$\zeta_{2;B}^E$	ζ_2^F	ζ_2^G	ζ_2^H	$\zeta_{2;\pi}^K$	$\zeta_{2;\Delta}^K$	$\zeta_{2;\pi}^L$	$\zeta_{2;\Delta}^L$
1	-18.99	-35.73	7.96	-4.21	-19.37	16.07	6.91	1.87	3.70	3.70	13.53	20.62	-4.95	0
2	-5.15	-9.79	2.23	-1.13	-5.44	5.89	2.34	0.70	1.39	1.39	3.98	5.33	-1.88	0
3	-1.46	-2.80	0.65	-0.32	-1.59	2.03	0.77	0.24	-0.10	-0.10	1.20	1.38	-0.66	0
n	ξ_2^D	$\xi_{2;R}^E$	$\xi_{2;B}^E$	ξ_2^F	ξ_2^G	ξ_2^H	$\xi_{2;\pi}^K$	$\xi_{2;\Delta}^K$	$\xi_{2;\pi}^L$	$\xi_{2;\Delta}^L$				
1	-8.51	-7.27	-1.88	-0.94	-1.88	-1.88	9.38	-2.12	0	2.74				
2	-2.54	-1.88	-0.49	-0.24	-0.49	-0.49	3.08	-0.55	0	0.71				
3	-0.78	-0.49	-0.12	-0.06	-0.13	-0.13	1.84	-0.14	0	0.18				
n	ζ_3^A	$\zeta_{3;R}^B$	$\zeta_{3;B}^B$	ζ_3^C	$\zeta_{3;R}^E$	$\zeta_{3;B}^E$	ζ_3^F	ζ_3^G	ζ_3^H	$\zeta_{3;\pi}^K$	$\zeta_{3;\Delta}^K$	$\zeta_{3;\pi}^L$	$\zeta_{3;\Delta}^L$	ζ_3^M
1	-3.59	7.78	-15.60	-10.18	16.61	-1.18	1.47	11.79	11.79	-9.21	-18.48	17.94	0	24.09
2	-1.06	2.14	-4.41	-2.71	7.41	-0.19	0.51	4.06	4.06	-2.88	-4.50	8.99	0	6.95
3	-0.32	0.62	-1.30	-0.76	2.78	-0.02	0.17	1.35	1.35	-0.90	-1.24	3.51	0	2.06
n	$\xi_{3;R}^E$	$\xi_{3;B}^E$	ξ_3^F	ξ_3^G	ξ_3^H	$\xi_{3;\pi}^K$	$\xi_{3;\Delta}^K$	$\xi_{3;\pi}^L$	$\xi_{3;\Delta}^L$					
1	-14.54	-0.94	-0.47	-2.90	-2.90	-5.73	2.09	0	-21.13					
2	-3.76	-0.24	-0.12	-0.75	-0.75	-1.97	0.54	0	-5.46					
3	-0.97	-0.06	-0.03	-0.19	-0.19	-0.65	0.14	0	-1.41					
n	ζ_+^A	ζ_+^D	$\zeta_{+;B}^E$	ζ_+^F	$\zeta_{+;\pi}^K$	$\zeta_{+;\Delta}^K$	$\zeta_{+;\pi}^L$	$\zeta_{+;\Delta}^L$						
1	1.04	-0.93	0.22	0.47	2.51	3.79	0.54	0						
2	0.29	-0.23	0.15	0.18	0.91	0.98	0.22	0						
3	-0.08	-0.06	0.06	0.06	0.31	0.25	0.08	0						
n	ξ_+^D	$\xi_{+;B}^E$	ξ_+^F	$\xi_{+;\pi}^K$	$\xi_{+;\Delta}^K$	$\xi_{+;\pi}^L$	$\xi_{+;\Delta}^L$							
1	-0.80	-0.47	-0.24	6.62	-1.24	0	-0.35							
2	-0.24	-0.12	-0.06	2.05	-0.32	0	-0.09							
3	-0.19	-0.04	-0.02	0.85	-0.08	0	-0.02							
n	ζ_-^A	ζ_-^C	ζ_-^D	$\zeta_{-;B}^E$	ζ_-^F	$\zeta_{-;\pi}^K$	$\zeta_{-;\Delta}^K$	$\zeta_{-;\pi}^L$	$\zeta_{-;\Delta}^L$					
1	-6.00	-11.95	-10.18	1.39	2.77	7.81	0	-3.45	0					
2	-1.69	-3.12	-2.58	0.63	1.04	2.29	0	-1.00	0					
3	-0.50	-0.86	-0.67	0.25	0.37	0.69	0	-0.30	0					
n	ξ_-^D	$\xi_{-;B}^E$	ξ_-^F	$\xi_{-;\pi}^K$	$\xi_{-;\Delta}^K$	$\xi_{-;\pi}^L$	$\xi_{-;\Delta}^L$							
1	-7.49	-1.41	-1.41	2.09	0.29	0	-0.31							
2	-1.85	-0.36	-0.36	0.65	0.07	0	-0.07							
3	-0.69	-0.11	-0.10	0.47	0.02	0	-0.01							

Table 2: $Q^2=0.04 \text{ (Gev/c)}^2$

n	ζ_1^A	$\zeta_{1;R}^B$	$\zeta_{1;B}^R$	ζ_1^C	ζ_1^D	$\zeta_{1;R}^E$	$\zeta_{1;B}^E$	ζ_1^F	ζ_1^G	ζ_1^H	$\zeta_{1;\pi}^K$	$\zeta_{1;\Delta}^K$	$\zeta_{1;\pi}^L$	$\zeta_{1;\Delta}^L$
1	23.67	39.89	-13.59	5.27	15.63	-18.35	-6.41	1.02	-4.10	-4.10	-10.14	2.17	9.98	0
2	6.44	10.86	-3.70	1.39	4.56	-6.52	-1.73	0.37	-1.49	-1.49	-2.11	0.54	3.71	0
3	1.83	3.09	-1.06	0.11	1.37	-2.19	-0.49	0.13	0.11	0.11	-0.46	0.14	1.27	0
n	ξ_1^D	$\xi_{1;R}^E$	$\xi_{1;B}^E$	ξ_1^F	ξ_1^G	ξ_1^H	$\xi_{1;\pi}^K$	$\xi_{1;\Delta}^K$	$\xi_{1;\pi}^L$	$\xi_{1;\Delta}^L$				
1	4.01	7.55	-0.94	-0.47	1.88	1.88	-0.71	-3.22	0	-5.38				
2	1.20	1.89	-0.24	-0.12	0.47	0.47	-0.21	-0.80	0	-1.35				
3	0.37	0.47	-0.06	-0.03	0.12	0.12	-0.06	-0.20	0	-0.01				
n	ζ_2^A	$\zeta_{2;R}^B$	$\zeta_{2;B}^B$	ζ_2^C	ζ_2^D	$\zeta_{2;R}^E$	$\zeta_{2;B}^E$	ζ_2^F	ζ_2^G	ζ_2^H	$\zeta_{2;\pi}^K$	$\zeta_{2;\Delta}^K$	$\zeta_{2;\pi}^L$	$\zeta_{2;\Delta}^L$
1	-21.72	-39.89	13.59	-5.27	-21.44	18.35	7.62	2.07	4.10	4.10	15.84	20.53	-6.96	0
2	-5.84	-10.86	3.70	-1.39	-6.00	6.52	2.53	0.75	1.49	1.49	4.56	5.13	-2.54	0
3	-1.64	-3.09	1.06	-0.11	-1.75	2.19	0.82	0.26	-0.11	-0.11	1.35	1.28	-0.86	0
n	ξ_2^D	$\xi_{2;R}^E$	$\xi_{2;B}^E$	ξ_2^F	ξ_2^G	ξ_2^H	$\xi_{2;\pi}^K$	$\xi_{2;\Delta}^K$	$\xi_{2;\pi}^L$	$\xi_{2;\Delta}^L$				
1	-9.84	-7.55	-1.89	-0.95	-1.88	-1.88	10.33	-2.14	0	3.37				
2	-2.92	-1.89	-0.49	-0.24	-0.49	-0.49	3.33	-0.54	0	0.84				
3	-0.89	-0.47	-0.12	-0.06	-0.13	-0.13	1.35	-0.13	0	0.21				
n	ζ_3^A	$\zeta_{3;R}^B$	$\zeta_{3;B}^B$	ζ_3^C	$\zeta_{3;R}^E$	$\zeta_{3;B}^E$	ζ_3^F	ζ_3^G	ζ_3^H	$\zeta_{3;\pi}^K$	$\zeta_{3;\Delta}^K$	$\zeta_{3;\pi}^L$	$\zeta_{3;\Delta}^L$	ζ_3^M
1	-6.77	10.32	-21.65	-10.86	19.41	-1.09	1.63	13.04	13.04	-10.76	-18.35	22.36	0	26.47
2	-1.90	2.80	-6.00	-2.86	8.20	-0.17	0.55	4.38	4.38	-3.21	-4.59	10.33	0	7.54
3	-0.55	0.80	-1.73	-0.23	2.97	-0.02	0.18	1.43	1.43	-0.97	-1.15	3.87	0	2.23
n	$\xi_{3;R}^E$	$\xi_{3;B}^E$	ξ_3^F	ξ_3^G	ξ_3^H	$\xi_{3;\pi}^K$	$\xi_{3;\Delta}^K$	$\xi_{3;\pi}^L$	$\xi_{3;\Delta}^L$					
1	-15.11	-0.94	-0.47	-2.61	-2.61	-6.18	1.81	0	-22.47					
2	-3.78	-0.24	-0.12	-0.65	-0.65	-2.08	0.45	0	-5.61					
3	-0.94	-0.06	-0.03	-0.12	-0.12	-0.68	0.11	0	-1.40					
n	ζ_+^A	ζ_+^D	$\zeta_{+;B}^E$	ζ_+^F	$\zeta_{+;\pi}^K$	$\zeta_{+;\Delta}^K$	$\zeta_{+;\pi}^L$	$\zeta_{+;\Delta}^L$						
1	0.33	-0.97	0.20	0.52	3.22	6.71	0.61	0						
2	0.10	-0.24	0.13	0.19	1.12	3.87	0.23	0						
3	0.03	-0.06	0.06	0.06	0.37	0.42	0.08	0						
n	ξ_+^D	$\xi_{+;B}^E$	ξ_+^F	$\xi_{+;\pi}^K$	$\xi_{+;\Delta}^K$	$\xi_{+;\pi}^L$	$\xi_{+;\Delta}^L$							
1	-0.98	-0.47	-0.47	-1.31	6.70	0	-0.36							
2	-0.29	-0.12	-0.12	-0.33	2.16	0	-0.09							
3	-0.09	-0.03	-0.03	-0.08	0.74	0	-0.02							
n	ζ_-^A	ζ_-^C	ζ_-^D	$\zeta_{-;B}^E$	ζ_-^F	$\zeta_{-;\pi}^K$	$\zeta_{-;\Delta}^K$	$\zeta_{-;\pi}^L$	$\zeta_{-;\Delta}^L$					
1	-10.49	-14.75	-10.65	1.43	3.08	8.87	0	-3.82	0					
2	-2.83	-3.80	-2.65	0.64	1.12	2.62	0	-1.11	0					
3	-0.80	-1.03	-0.69	0.23	0.38	0.80	0	-0.33	0					
n	ξ_-^D	$\xi_{-;B}^E$	ξ_-^F	$\xi_{-;\pi}^K$	$\xi_{-;\Delta}^K$	$\xi_{-;\pi}^L$	$\xi_{-;\Delta}^L$							
1	-8.58	-1.42	-1.42	2.54	0.27	0	-0.30							
2	-2.55	-0.35	-0.35	0.78	0.07	0	-0.08							
3	-0.78	-0.08	-0.08	0.34	0.02	0	-0.02							

Table 3: $Q^2=0.06 \text{ (Gev/c)}^2$

Although we cannot pin down the values of unknown parameters such as $G_1, G_2 \dots$ since so far no amplitudes directly extracted from experiment are available, we still can make some observation on these integrated quantities which may shed the lights to the physics behind them.

Firstly, the values of $\zeta_i^A, \zeta_{i;R}^B$ and $\zeta_{i;R}^E$ are significant larger among the ones of tree diagrams because they associate with the $\mathcal{O}(\epsilon^2)$ amplitudes and others are related to $\mathcal{O}(\epsilon^3)$ amplitudes (except $\zeta_{3;R}^B$ because they are identically zero if recoil effect is not included). Therefore power counting scheme is well preserved here and the wild behavior of amplitudes due to the pole of $\Delta(1232)$ resonance is tamed in the weighted integrals. The Q^2 dependences of these quantities of tree diagrams are less sensitive than ones of the loop diagrams, which is due to up to $\mathcal{O}(\epsilon^3)$ where there is no vertex proportional to Q^2 . The Q^2 dependences of these multipole results of tree diagrams are only from the propagators of the nucleon and $\Delta(1232)$. However if we continue to go to higher Q^2 range, power counting could not be kept without modification. We made this approximation on the propagators of the nucleon:

$$S(k) = \frac{i}{v \cdot k + \frac{(v \cdot k)^2 - k^2}{2m_p}} \sim \frac{i}{v \cdot k} + \frac{i((v \cdot k)^2 - k^2)}{2m_p(v \cdot k)^2}.$$

k is the four momentum of the nucleon. Here we assume that $Q^2 \sim \epsilon^2$. When Q^2 goes higher, *e.g.*, $Q^2 \sim \epsilon\Lambda \sim 0.3(\text{Gev}/c)^2$, obviously this expansion will break down or at least the counting rules have to be changed. One way to do this is to calculate them in full relativistic formulation then expand it by $\frac{\omega}{M_p}$ but not by Q^2 . So we have to limit ourselves in the region $Q^2 \leq 0.1 \sim 0.2(\text{Gev}/c)^2$.

In general the weighted integrals related to the loop diagrams, such as ζ_i^D, ζ_i^K and ζ_i^L , are all more sensitive to the change of Q^2 . Their Q^2 dependences are from the propagators of pions in the $N\pi$ and $\Delta\pi$ loops. Naively we might conclude that the effects of the pion cloud are quite significant in the Q^2 evolution of the weighted integrals. However these quantities are μ -dependent and their μ dependences are compensated by other quantities. Therefore it is difficult to identify the generic π cloud effects. On the other hand, the quantities such as $\zeta_{i;\Delta}^K$ which is free of any μ dependence are less sensitive to Q^2 . Such a quantity represents the interference between the imaginary part of the $N\pi$ loop and the δ function of the Δ propagator. Again the validity of our expansion is limited in the low Q^2 region because of the possible modification in the propagators of nucleon and $\Delta(1232)$ in the loops.

The longitudinal multipoles L_{1+} and L_{1-} are now the subjects of intense study [13, 15]. It was suggested that in parallel kinematics, the particular combination $4S_{1+} + S_{1-} - S_{0+}$ can be measured through the recoil polarizations of the nucleon and attempts have been made to extract the CMR= ImS_{1+}/ImM_{1+} at the $\Delta(1232)$ peak under the assumption that the nonresonant contribution is negligible. However, a recent measurement on P_n which should vanish if the background is really negligible at the peak, was reported as unexpectedly large

[10]. The MIT/Bates group also measured the longitudinal-transverse interference response R_{LT} and the related asymmetry A_{LT} at $W=1.172$ GeV, and show that available models all fail to explain the W -dependence of these observables.

Our approach gives no information on W dependence since we integrate through the $\Delta(1232)$ region. If more data is taken at a different energy in the Δ region, then our result can be verified because no new counterterms are needed for both L_{1+} and L_{1-} up to $\mathcal{O}(\epsilon^3)$. L_{1+} is dominated by the $\Delta \pi$ loop diagrams; on the other hand, the L_{1-} is dominated by the nucleon sector. There is one special interesting combination to note: $L_{1+} - E_{1+}$. It is exclusively due to the $\Delta\pi$ loop; further, it is a μ -independent quantity. It is interesting to see if any observable is particularly sensitive to such a combination [25].

A complete determination of counterterms in the $\gamma\pi N\Delta$ system can only be done by a more extensive study on more processes such as πN scattering or Compton scattering. For example, the contributions of $g_{\pi\Delta N}\tilde{G}_6$ cannot be disentangled with the ones of $\tilde{g}_{\pi\Delta N}\dot{G}_1$ in either photo- or electroproduction of pions. But only in $\tilde{g}_{\pi\Delta N}$ participates the πN scattering. Similarly only \dot{G}_1 and \tilde{G}_6 appear in the Compton scattering. Actually they share most of the counterterms and that is why even we could make no prediction on any physical observables. Still our approach has predictive power. A similar calculation on πN elastic scattering is proceeding [25].

Finally, we emphasized that in order to compare with our results, we require these double polarization experiments continuously run for a wide range of energy and eventually to extract the individual amplitudes. To describe both longitudinal and transverse amplitudes of pion electroproduction in the resonance region including the resonant and background contributions are a formidable task. Our approach seem to be a very promising way, at least in the low Q^2 region.

5 Conclusion

In summary, we calculated the amplitudes of pion electroproduction up to third order in the framework of HBChPT including explicit Δ (1232) isobars. The Q^2 dependences of the weighted integrals of these amplitudes are presented here and they turn out to be good testing grounds for this phenomenological extension of HBChPT because all the counterterms are fixed at $Q^2=0$. The predictive power of the method proposed in [1] was shown. Our final goal is to treat various processes such as $\gamma N \rightarrow \gamma N$, $\gamma N \rightarrow \gamma N$ and $\pi N \rightarrow \pi N$ in the $\Delta(1232)$ region in a unified, consistent framework, and determine all counterterms from experiments without any model dependence, which requires calculations on more processes and the results of multipoles analyses extracted from experimental data.

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